General Balance and the Multi-Stratum ANOVA

By Curt Lee

Summary: Then concept of **General Balance** was first defined by John Nelder. The philosophy behind it requires that the structure of treatments is specified separately from the dispersion structure. This concept is implemented in GenStat's ANOVA algorithm which partitions the total sum of squares into components known as **strata**, one for each error term. Each stratum contains the sums of squares for the treatment terms estimated between the units of the **stratum** which represent the random variability of the stratum. For designs with several error terms, a Multi-Stratum ANOVA is produced. This approach results in an analysis that matches the design of a field experiment.

General balance in experimental design was first defined by Nelder. The class of generally balanced designs covers a wide range of designs with one or several error terms. The philosophy behind it is unlike other theories of experimental design and require the structure and treatments to be specified separately from the dispersion structure. The concept of general balance is of special interest to those who use designs with several error terms. The opinion of some is that a lack of interest in general balance has limited the skill level of many experimenters. My observation is that that the idea of general balance simplifies the teaching of this subject matter and increases the competency and efficiency for those learning it and using it in practice.

The concept of general balance is implemented in GenStat's ANOVA algorithm which works by an efficient sequence of **sweeps**. This algorithm is very efficient and thus has a computational advantage. Computational simplicity of general balance may have little to do with practical experiments in these days of high computer power but it does aid in interpretation. A design which is generally balanced with respect to meaningful contrasts may be superior to a technically optimal design (Bailey, 1993).

The **Multi-stratum analysis of variance** is a leading principle behind the analysis agricultural data and is fundamental to understanding design itself. This tradition in design and analysis is taught at Rothamsted Research. A recent book, "Statistical methods in biology", gives a detailed explanation of this approach (Welham, 2015).

In a statistical way of speaking, we structure our trials into **strata** to minimize the heterogeneity of error within blocks. We may further structure our trials to accommodate equipment used to apply treatments. Consequently, restrictions are imposed on layout of an experiment every time we design and conduct an experiment. These restrictions create different structural sources of variability among the experimental units called strata. Each restriction in the structure of an experiment is called a **stratum**.

The multi-stratum ANOVA accounts for the physical structure of the experimental material or blocking imposed by the experimenter. It is an analysis approach that creates an ANOVA table with separates components for each stratum defined by the structural component. The variation within each stratum is partitioned into the sums of squares associated with the treatments that vary between the units at that level of the design and a residual term. The great advantage of the multi-stratum ANOVA is the recognition of the interplay between blocking and treatment structure so that treatment effects are always allocated to the correct strata so appropriate variance are calculated. There is an old adage in statistics, "as the randomization is, so should the analysis be" (Pearce, 1988). This is a natural approach to the analysis of data from agricultural field experiments. Very few software packages are available that create multi-stratum ANOVA tables.

The Genstat ANOVA is not without limitations. It can only be formed when the explanatory and structural component obey certain conditions of **general balance**. The properties of general balance are that the block terms are mutually orthogonal, the treatment terms are mutually orthogonal, and contrast of each treatment terms all have equal efficiency factors in each of the strata where they are estimated (Payne, 1998).

Although GenStat implicitly identifies terms in the structural component of the model as random, they are calculated by least square estimates as if they were fixed terms. Consequently, the multi-stratum ANOVA is a **fixed effects model**. The long and short of the multi-stratum ANOVA is that if you've specified the structure correctly then treatment terms get tested at the correct level of structure. If you don't trust software, or are not using a multi-stratum ANOVA table, by all means working out estimated means squares then becomes an essential part of the process (S.J. Welham, personal, communication, 2015).

My opinion is that the GenStat ANOVA, with its multi-stratum ANOVA, should always be the starting point for the analysis of data and is my go-to method for checking the output of other statistical software. At times, it is useful for checking the appropriateness of a more complex analysis (i.e. did the analysis fit the design).

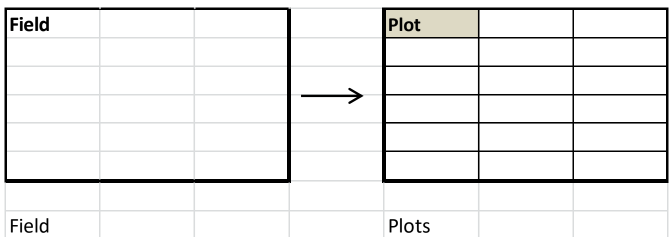
STRATA in a Field Experiment

Think of strata in terms of structural restrictions imposed on the experimental units in a field.

CRD

Field ⟶ Plots

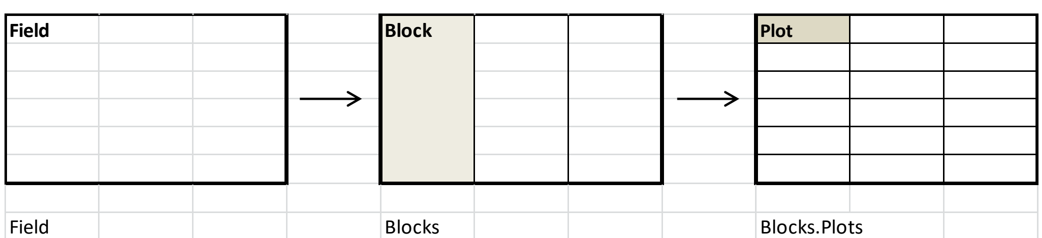
(this has no structural restrictions imposed)



There is **no stratum**.

RCBD and RCBD Factorial

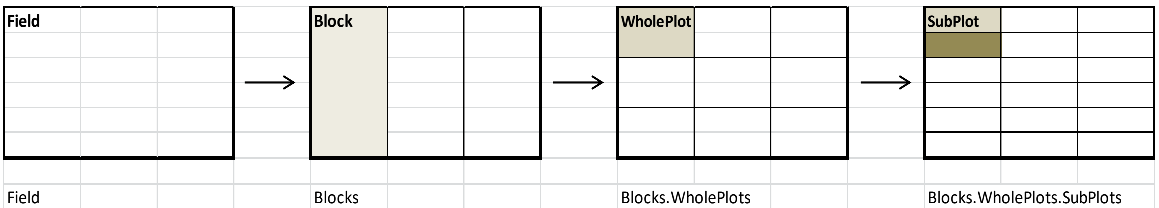
Field ⟶ Blocks ⟶ Plots



There are ***two stratum***:
1. Blocks (Variation between blocks)
2. Blocks.Plots (Variation between plots within block)

Split Plot

Field → Blocks → WholePlots → SubPlots

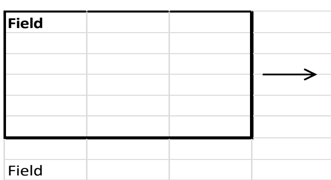


There are ***three stratum***:

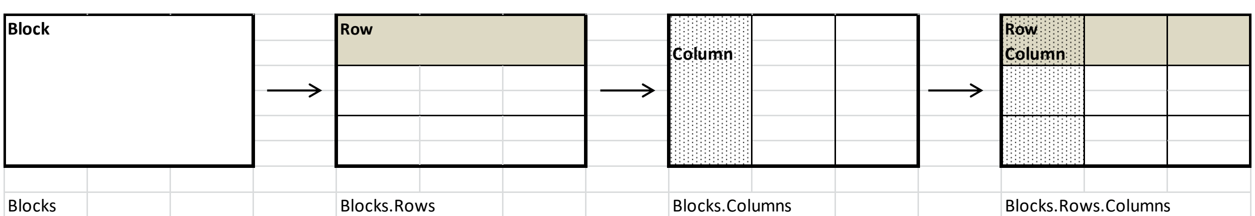
1. Blocks
2. Blocks.WholePlot
3. Blocks.WholePlots.SubPlots

Strip Plot

Field →



Blocks → Blocks.Rows → Blocks.Columns → Blocks.Row.Columns



There are ***four stratum***:

1. Blocks
2. Blocks.Rows
3. Blocks.Columns
4. Blocks.Rows.Columns

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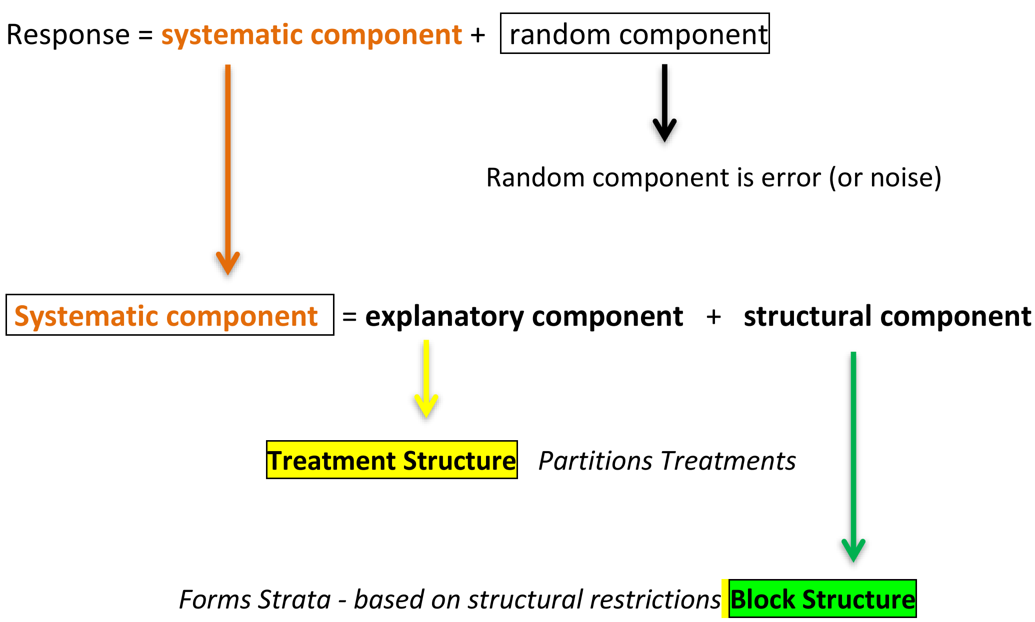
Construction of Statistical Models

We can define a model based on explanatory and structural components.

For example,

Yield = systemic component + random component

The random component is ***error***. The systemic component is comprised of two parts, the **explanatory** and **structural components**, as per the following diagram.

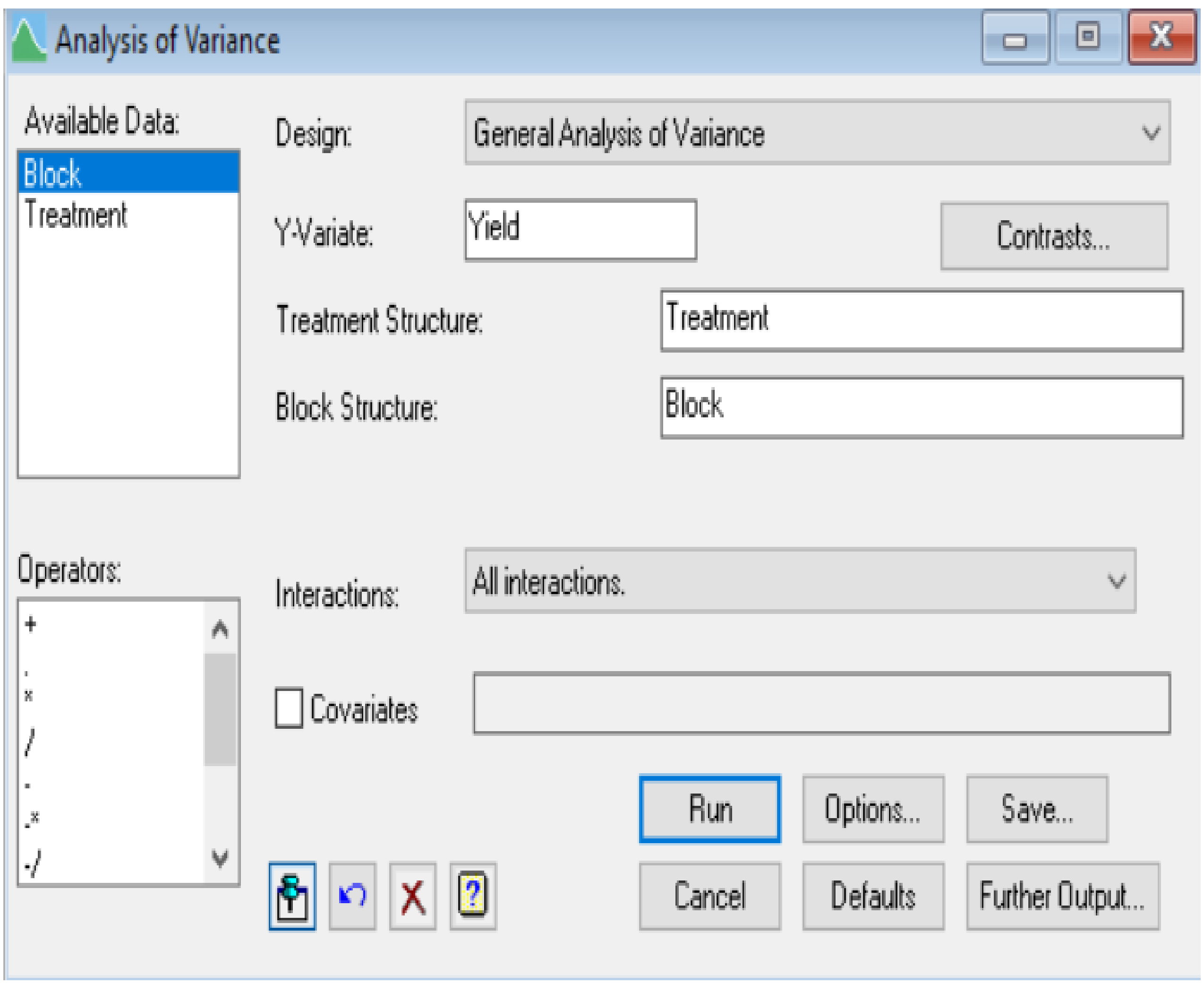


The **treatment model** is defined by the TREATMENTSTRUCTURE directive which specifies treatment model terms to be fitted by ANOVA. The **block model** is defined by the BLOCKSTRUCTURE directive, which specifies the underlying *blocking and randomization* structure (strata) of a design that is to be analyzed by ANOVA.

This concept directly translates into the **GenStat** model through the graphical user interface (GUI). The GUI allows you to directly analyze the data by using simple block and treatment structure. An RCB example is as follows.

Yield = **treatment structure** + **block structure** + error structure

Y-Variate = **Yield**		
Explanatory component =	**Treatment Structure**	*Partitions Treatments*
Structural component =	**Block Structure**	*Forms Strata - based on structural restrictions*



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Deriving a more complicated Model Formula

Model formula are derived through a combination of identifiers (terms) and operators. The operators proved a convenient way of stating a model in a compact form. The two most common relationships between terms (factors) are **nested** and **crossed structures**. Below is an example of the operators used for such a relationship.

The **/ (forward slash)** operator indicates a **nested relationship**.

This is a hierarchical relationship where multiple units of one structural level are entirely contained within a unit at a higher level.

Block/plot = Block + Block.Plot (Blocks and plots within blocks)

The * (star) operator indicates **a crossed relationship**.

Variety * Fertilizer = Variety + Nitrogen + Variety.Nitrogen

Commonly used Operators

Addition operator (**+**)	**A+B+C** main effects of **A, B**, and **C**
Interaction operator (**.**)	**A.B** interaction of **A** and **B**
Crossing operator (*****)	**A*B** is equivalent to **A+B+A.B**
Nesting operator (**/**)	**A/B** is equivalent to **A+A.B**

STRUCTURAL AND EXPLANATORY COMPONENT EXAMPLES

The following are some common examples used in agriculture.

Example:	Structural Components	Explanatory Component
CRD:	None used	Treatment
RCBD:	Block/Plot	Treatment
Latin Square:	Row*Column	Variety
Split Plot:	Block/W_Plot/S_Plot	Variety*Nitrogen
Strip Plot:	Block/(W_Plot1*W_Plot2)	Nitrogen*Variety
Split Split Plot:	Block/W_Plot/S_Plot/SS_Plot	Nitrogen*Management*Variety
Strip-Split Plot:	Block/(Row*Column)/PlantingMethod	Variety*Nitrogen*PlantingMethod

One of Genstat's noted achievements is that it incorporated John Nelder's theory of balance into Graham Wilkinson algorithm, and pushed this concept to the limit. In summary, it puts all the work of Fisher, Yates and Finney into a single framework so that any design can be described in terms of two formulas. This made it possible to retain the conceptual simplicity of ANOVA type strata in the analysis, which is very intuitive for those analyzing designed experiments. This approach matches the allocation procedure to the analysis. The randomization carried out guides and analysis and the analysis you intend guides the randomization. (Senn, 2019).

SIMPLE and MULTI-STRATUM ANOVA Tables

A comparison of a simple ANOVA table and GenStat's Multi-Stratum ANOVA which divides the ANOVA table into strata.

Analysis of variance

A **simple ANOVA** table does not make any distinction between describing the underlying structure of the data and those indicating the treatments applied.

Variate: Yield

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Block	3	1944361.	648120.	5.86	
Treatment	5	1198331.	239666.	2.17	0.113
Residual	15	1658376.	110558.		
Total	23	4801068.			

The **multi-stratum ANOVA** table for the RCBD rearranges the simple ANOVA table to reflect the structure of the experiment. The RCBD has two distinct strata, a **Block stratum** and a **Block.Plot stratum**.

Variate: Yield

Source of variation	d.f.	s.s.	m.s.	v.r.	F pr.
Block stratum	3	1944361.	648120.	5.86	
Block.Plot stratum					
Treatment	5	1198331.	239666.	2.17	0.113
Residual	15	1658376.	110558.		
Total	23	4801068.			

The multi-stratum ANOVA table is a general ANOVA table that preserves the distinction between the terms describing the underlying variability structure of the data (block structure) and those indicating the treatments applied (treatment structure).

MULTI-STRATUM ANOVA’S and INCORPATING TREATMENT STRUCTURE

When analyzing data, we emphasize the structure of the experiment which is defined by the correct structural model (**block structure**). We also can examine ways of translating questions about the set of treatments into the statistical analysis which can be directly answered by an F-Test within the ANOVA table. These questions are defined in the treatment model (**treatment structure**). Examples of forming multi-stratum ANOVA’s and incorporating treatment structure are given in the following examples

Example 1. ANOVA for Potato yield data (Welhelm, 2015)

An ANOVA is completed for a potato trial with an RCB design (Data set 1). It has **two strata**, a **Block stratum** with corresponds to variation between blocks and a **Block.Plot stratum** which correspond to variation between plots within blocks.

The ANOVA indicates a significant Fungicide effect, but this includes the comparison with the control. We would expect an effect, but this does not tell us what was different. Not very useful information other than yes, we have difference. The question at hand is did this analysis account for all structural sources of variation and can explanatory component (treatment structure) be partitioned into more meaningful comparisons about the fungicides.

Explanatory component: Fungicide
Structural component: Block/Plot

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3     14987.      4996.    1.43

Block.Plot stratum
Fungicide                  4    133419.     33355.    9.58  0.001
Residual                  12     41797.      3483.

Total                     19    190203.
```

Example 2. ANOVA for Potato yield data in which yields were collected from individual rows within a plot (Welham, 2015).

This AVOVA can be further expanded if we account for an additional source of variation (Rows). Data set 2 contains yield data for individually harvest rows in the trial. This analysis contains **3 stratum**, a **Block stratum** with corresponds to variation between blocks. The **Block.Plot stratum** which correspond to variation between plots within blocks, and the **Block.Plot.Row stratum** which corresponds to variation between rows within a plot, plots within a Block. Row yields are from data points from subsampling within plots, which was ignored in the first analysis.

While the F test and conclusion remain the same, without taking into account the subsampling (rows), Block and Fungicide variance ratios are inflated, treatment SEM's, SED's and LSD's are underestimated.

Explanatory component: **Fungicide**
Structural component: **Block/Plot/Row**

```
Analysis of variance
====================

Variate: RowYield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3     59949.     19983.    1.43

Block.Plot stratum
Fungicide                  4    533677.    133419.    9.58  0.001
Residual                  12    167187.     13932.    4.47

Block.Plot.Row stratum    60    186848.      3114.

Total                     79    947661.
```

Example 3. Potato yield data with partitioning of treatments to compare control and treated, and among treatments (Welham, 2015).

Below the Multi-Stratum ANOVA is partitioned to compare control versus treated (Type) and variation among fungicide treatments (Type.Fungicide)

From the ANOVA we can conclude that that control is significantly different form the treated (Type). Also, we conclude there is no difference between fungicide treatments.

Explanatory component: **Type+Type.Fungicide** *or Type/Fungicide*
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3     14987.      4996.    1.43

Block.Plot stratum
Type                       1    125294.    125294.   35.97  <.001
Type.Fungicide             3      8125.      2708.    0.78  0.529
Residual                  12     41797.      3483.

Total                     19    190203.
```

Example 4. Potato yield data with treatments partitioned into orthogonal contrast to compare mode of action of fungicides (Welham, 2015).

The multi-stratum ANOVA can be further partitioned into orthogonal comparisons. In this example, we want to compare fungicide mode of action. F1 and F4 (A) are one mode of action while F2 and F3 (B) are another mode of action. From the ANOVA, we conclude there are no differences between the fungicides modes of action.

Explanatory component: **COMP(Fungicide;1;Cont)**
Structural component: **Block/Plot**

```
Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3     14987.      4996.    1.43

Block.Plot stratum
Fungicide                  4    133419.     33355.    9.58  0.001
  Contrast: Mode A vs B    1      5402.      5402.    1.55  0.237
Residual                  12     41797.      3483.

Total                     19    190203.
```

Example 5. Potato yield data with treatments partitioned into orthogonal contrast to compare control versus fungicide treatments, mode of action of fungicides, and fungicides within modes of action. (Welham, 2015).

The multi-stratum ANOVA can be further partitioned into orthogonal comparisons. In this example we want to compare control versus fungicides, fungicide mode of action, F1 versus F4, and F2 versus F3.

Explanatory component: **COMP(Fungicide;4;Cont_1)**
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3     14987.      4996.    1.43

Block.Plot stratum
Fungicide                  4    133419.     33355.    9.58  0.001
  Control vs Fungicide     1    125294.    125294.   35.97  <.001
  Mode A vs. Mode B        1      5402.      5402.    1.55  0.237
  F1 vs. F4                1      2178.      2178.    0.63  0.444
  F2 vs. F3                1       544.       544.    0.16  0.699
Residual                  12     41797.      3483.

Total                     19    190203.
```


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Example 6. Forage crop yields with Nitrogen treatments (Welham, 2015).

Explanatory component: **N**
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              2     2.8385     1.4192    4.40

Block.Plot stratum
N                          3     6.1434     2.0478    6.35  0.027
Residual                   6     1.9359     0.3227

Total                     11    10.9178
```

Forage crop yields with Nitrogen treatments partitioned into **polynomial contrasts** (linear, quadratic and cubic**)**. Nitrogen rate has a significant linear response. Note that a linear trend dominates the pattern (F = 18.37, P>0.005). There is no evidence of a quadratic trend or a higher order trend as indicated by deviations. The **deviations** term represents the variation of a set of treatment effects that has not been explained by a fitted set of *contrasts.*

Explanatory component: **POL(N;3)**
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              2     2.8385     1.4192    4.40

Block.Plot stratum
N                          3     6.1434     2.0478    6.35  0.027
  Lin                      1     5.9283     5.9283   18.37  0.005
  Quad                     1     0.0085     0.0085    0.03  0.876
  Deviations               1     0.2065     0.2065    0.64  0.454
Residual                   6     1.9359     0.3227

Total                     11    10.9178
```

Example 7. Canola example (Agro-Tech).

Below is a comparison of analysis of a canola trial before and after incorporating different strata in the structural component and partitioning the explanatory component for a canola variety trial completed in 2014. The first analysis ignores all underlying treatment and block structure while the second analysis accounts for a simple block structure

Explanatory component: **TRT**
Structural component:

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.
TRT                        8   2616141.    327018.   13.95  <.001
Residual                  27    632843.     23439.
Total                     35   3248984.
```

Explanatory component: **TRT**
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3    174554.     58185.    3.04

Block.PLOT stratum
TRT                        8   2616461.    327058.   17.10  <.001
Residual                  24    459068.     19128.

Total                     35   3250084.
```

The third analysis accounts for the actual treatment structure (crossed) but ignores the underlying block structure (nesting). Note that the variation (65%) is from Variety (2138952/3250084). The v.r. (F value) for variety is 55.91, compared to 5.31 for harvest method and 3.59 for the Variety x Harvest Method interaction. So even though the interaction is significant, most of the variation is from variety.

Explanatory component: **Variety*Harvest Method**
Structural component: **Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3    174554.     58185.    3.04

Block.PLOT stratum
Variety                    2   2138952.   1069476.   55.91  <.001
Harvest_Method             2    203119.    101560.    5.31  0.012
Variety.Harvest_Method     4    274390.     68598.    3.59  0.020
Residual                  24    459068.     19128.

Total                     35   3250084.
```

The final analysis accounts for the actual crossed explanatory component (treatment structure) and actual structural component (block structure). This trial was a **split plot**.

Explanatory component: **Variety*Harvest Method**
Structural component: **Block/WholePlot/SubPlot**

Expands to: Block + Block.WholePlot + Block.Wholeplot.SubPlot

```
Analysis of variance
====================

A Split plot has three strata, a Block stratum and a Block.WholePlot stratum,
and a Block.WholePlot.SubPlot stratum.


Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              3    174554.     58185.    2.24

Block.WholePlot stratum
Variety                    2   2138952.   1069476.   41.17  <.001
Residual                   6    155847.     25974.    1.54

Block.WholePlot.SubPlot stratum
Harvest_Method             2    203119.    101560.    6.03  0.010
Variety.Harvest_Method     4    274390.     68598.    4.07  0.016
Residual                  18    303221.     16846.

Total                     35   3250084.
```

Example 8. A CRD Fertilizer trial comparing sources, levels, and control versus treated (IRRI).

For structured experiments, multiple comparison procedure is inappropriate and partitioning of the treatment effects is required to test specific comparisons that were planned. In this case we test control versus treated, comparison between sources, comparisons between levels, and interaction of levels and sources. Note that this is a CRD, so it had no underlying structural component (no strata) but is analyzed as a RCBD for this example

Explanatory component: Treatment
Structural component: Rep

```
Analysis of variance
====================

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

REP stratum                2     0.3432     0.1716    0.56

REP.*Units* stratum
TREATMENT                  8    19.8485     2.4811    8.13  <.001
Residual                  16     4.8843     0.3053

Total                     26    25.0760
```

Explanatory component: Control_vs_Treated/(Source*Level)
Structural component: Rep

```
Analysis of variance
====================

The treatment source of variations has been partitioned into control
versus treated, comparison between treatment sources, comparisons
between treatment levels, and interaction of levels and sources

Variate: GYIELD

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

REP stratum                2     0.3432     0.1716    0.56

REP.*Units* stratum
Control_vs_Treated         1    11.4615    11.4615   37.55  <.001
Control_vs_Treated.Source
                           3     3.4439     1.1480    3.76  0.032
Control_vs_Treated.Level
                           1     4.1921     4.1921   13.73  0.002
Control_vs_Treated.Source.Level
                           3     0.7509     0.2503    0.82  0.502
Residual                  16     4.8843     0.3053

Total                     26    25.0760
```

Example 9. Barley and oat trial comparing beta-glucan content (Lee thesis). In this analysis, no differences are found between barley and oat.

Explanatory component: Crop
Structural component: Block

```
Analysis of variance
====================

Variate: %_beta_glucan

Source of variation     d.f.        s.s.        m.s.    v.r.  F pr.

Block stratum              2      0.1851      0.0925    0.20

Block.*Units* stratum
Crop                       1      0.0991      0.0991    0.21  0.649
Residual                  53     25.0664      0.4730

Total                     56     25.3506
```

Explanatory component: Crop/(Within_Barley+Within_Oat)
Structural component: Block

```
Analysis of variance
====================
```

Crop source of variation has been further partitioned into the comparisons within barley (comparing barley varieties) and within oat (comparing oat varieties). Differences are detected within barley varieties and within oat varieties.

```
Variate: %_beta_glucan

Source of variation     d.f.        s.s.        m.s.    v.r.  F pr.

Block stratum              2     0.18506     0.09253    1.34

Block.*Units* stratum
Crop                       1     0.09912     0.09912    1.43  0.239
Crop.Within_Barley         8    14.96201     1.87025   27.07  <.001
Crop.Within_Oat            9     7.61715     0.84635   12.25  <.001
Residual                  36     2.48728     0.06909

Total                     56    25.35061
```


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Example 10. Wheat 3 factor split plot (Agro-Tech). Whole plots are reduced and standard fertility. Whole plots are divided into four split plots, early timing no fungicide, early timing fungicide, late timing no fungicide, late timing fungicide.

Explanatory component: Fertility*Fungicide*Timing
Structural component: Block/W.Plot/S.Plot

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Block stratum              5     659.77     131.95    4.26

Block.W_Plot stratum
Fertility                  1    1328.21    1328.21   42.85  0.001
Residual                   5     154.99      31.00    1.10

Block.W_Plot.S_Plot stratum
Fungicide                  1     649.25     649.25   23.01  <.001
Timing                     1     111.55     111.55    3.95  0.056
Fertility.Fungicide        1      47.05      47.05    1.67  0.206
Fertility.Timing           1       5.45       5.45    0.19  0.663
Fungicide.Timing           1     155.42     155.42    5.51  0.026
Fertility.Fungicide.Timing
                           1       0.50       0.50    0.02  0.895
Residual                  30     846.49      28.22

Total 47     3958.67
```

Example 11a. Analysis of combined randomized complete block experiments (Bowley). Two locations (Elora and Thunder Bay) of a randomized complete block experiment are combined for analysis

There are **two strata**. The Location.Block **stratum** with corresponds to variation between Blocks within locations. The **Location.Block.Plot stratum** which corresponds to variation between Plots within blocks within locations.

This is a fixed effects analysis, thus Blocks and Locations are considered a fixed effect.

Explanatory component: **Location*Entry**
Structural component: **Location. Block/Plot**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Location.Block stratum
Location                   1    54.2145    54.2145   51.07  <.001
Residual                   6     6.3696     1.0616    3.58

Location.Block.Plot stratum
Entry                      6    12.4693     2.0782    7.01  <.001
Location.Entry             6     2.4643     0.4107    1.38  0.247
Residual                  36    10.6779     0.2966

Total                     55    86.1955
```


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Example 11b. Analysis of combined randomized complete block experiments (Bowley).

The treatment source of variations can further be partitioned into a location contrast (Elora versus Thunder Bay) and Entry contrast (Early versus Late)

Explanatory component: **Location*Entry**

Structural component: **COMP(Location;1;Cont_1)* COMP(Entry;1;Cont)**

```
Analysis of variance
====================

Variate: Yield

Source of variation     d.f.       s.s.       m.s.    v.r.  F pr.

Location.Block stratum
Location                   1    54.2145    54.2145   51.07  <.001
  Elora vs Thunder Bay     1    54.2145    54.2145   51.07  <.001
Residual                   6     6.3696     1.0616    3.58

Location.Block.Plot stratum
Entry                      6    12.4693     2.0782    7.01  <.001
  Early vs Late            1     2.7005     2.7005    9.10  0.005
Location.Entry             6     2.4643     0.4107    1.38  0.247
  Elora vs Thunder Bay.Early vs Late
                           1     1.6010     1.6010    5.40  0.026
Residual                  36    10.6779     0.2966

Total                     55    86.1955
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Appendix A.

Table 1. Potato yield data.

ID	Block	Plot	Type	Fungicide	Yield
1	1	1	Treated	F3	642
2	1	2	Control	Control	377
3	1	3	Treated	F2	633
4	1	4	Treated	F1	527
5	1	5	Treated	F4	623
6	2	1	Treated	F2	600
7	2	2	Control	Control	408
8	2	3	Treated	F3	708
9	2	4	Treated	F4	550
10	2	5	Treated	F1	604
11	3	1	Control	Control	500
12	3	2	Treated	F2	650
13	3	3	Treated	F3	662
14	3	4	Treated	F4	562
15	3	5	Treated	F1	606
16	4	1	Treated	F3	504
17	4	2	Treated	F2	567
18	4	3	Treated	F1	533
19	4	4	Control	Control	333
20	4	5	Treated	F4	667

Table 2. Potato yield data (with row yields).

ID	Block	Plot	Row	Fungicide	RowYield
1	1	1	1	F3	720
2	1	1	2	F3	528
3	1	1	3	F3	678
4	1	1	4	F3	642
5	1	2	1	Control	348
6	1	2	2	Control	405
7	1	2	3	Control	364
8	1	2	4	Control	391
9	1	3	1	F2	652
10	1	3	2	F2	658
11	1	3	3	F2	569
12	1	3	4	F2	653
13	1	4	1	F1	635
14	1	4	2	F1	512
15	1	4	3	F1	536
16	1	4	4	F1	425
17	1	5	1	F4	642
18	1	5	2	F4	639
19	1	5	3	F4	642
20	1	5	4	F4	569
21	2	1	1	F2	554
22	2	1	2	F2	618
23	2	1	3	F2	621
24	2	1	4	F2	607
25	2	2	1	Control	411
26	2	2	2	Control	374
27	2	2	3	Control	396
28	2	2	4	Control	451
29	2	3	1	F3	682
30	2	3	2	F3	741
31	2	3	3	F3	712
32	2	3	4	F3	697
33	2	4	1	F4	639
34	2	4	2	F4	544
35	2	4	3	F4	521
36	2	4	4	F4	496
37	2	5	1	F1	583
38	2	5	2	F1	530
39	2	5	3	F1	629
40	2	5	4	F1	674
41	3	1	1	Control	561

42	3	1	2	Control	491
43	3	1	3	Control	429
44	3	1	4	Control	519
45	3	2	1	F2	555
46	3	2	2	F2	633
47	3	2	3	F2	715
48	3	2	4	F2	697
49	3	3	1	F3	638
50	3	3	2	F3	712
51	3	3	3	F3	633
52	3	3	4	F3	665
53	3	4	1	F4	505
54	3	4	2	F4	597
55	3	4	3	F4	607
56	3	4	4	F4	539
57	3	5	1	F1	598
58	3	5	2	F1	620
59	3	5	3	F1	596
60	3	5	4	F1	610
61	4	1	1	F3	451
62	4	1	2	F3	493
63	4	1	3	F3	535
64	4	1	4	F3	537
65	4	2	1	F2	513
66	4	2	2	F2	626
67	4	2	3	F2	574
68	4	2	4	F2	555
69	4	3	1	F1	441
70	4	3	2	F1	467
71	4	3	3	F1	701
72	4	3	4	F1	523
73	4	4	1	Control	367
74	4	4	2	Control	319
75	4	4	3	Control	361
76	4	4	4	Control	285
77	4	5	1	F4	631
78	4	5	2	F4	618
79	4	5	3	F4	689
80	4	5	4	F4	730

Table 3. Forage yield response to Nitrogen treatments.

ID	Block	Plot	N	Yield
1	1	1	0	10.42
2	1	2	140	12.21
3	1	3	210	12.85
4	1	4	70	12.22
5	2	1	70	11.62
6	2	2	0	11.98
7	2	3	210	12.81
8	2	4	140	12.67
9	3	1	70	11.13
10	3	2	210	12.57
11	3	3	0	9.82
12	3	4	140	10.92

Table 4. Canola split plot yield trial in RCB design.

Block	WholePlot	SubPlot	TRT	Variety	Harvest Method	Yield
1	1	1	1	L140P	A (swathed)	3113
2	1	1	1	L140P	A (swathed)	3314
3	1	1	1	L140P	A (swathed)	3096
4	1	1	1	L140P	A (swathed)	2910
1	2	1	2	L120	A (swathed)	2699
2	2	1	2	L120	A (swathed)	2818
3	2	1	2	L120	A (swathed)	2771
4	2	1	2	L120	A (swathed)	2749
1	3	1	3	DKL 38-48	A (swathed)	2827
2	3	1	3	DKL 38-48	A (swathed)	2935
3	3	1	3	DKL 38-48	A (swathed)	2317
4	3	1	3	DKL 38-48	A (swathed)	2791
1	1	2	4	L140P	B (delayed swath)	3058
2	1	2	4	L140P	B (delayed swath)	3117
3	1	2	4	L140P	B (delayed swath)	3179
4	1	2	4	L140P	B (delayed swath)	3336
1	2	2	5	L120	B (delayed swath)	3001
2	2	2	5	L120	B (delayed swath)	2927
3	2	2	5	L120	B (delayed swath)	2720
4	2	2	5	L120	B (delayed swath)	3200
1	3	2	6	DKL 38-48	B (delayed swath)	2607
2	3	2	6	DKL 38-48	B (delayed swath)	2493
3	3	2	6	DKL 38-48	B (delayed swath)	2280
4	3	2	6	DKL 38-48	B (delayed swath)	2599
1	1	3	7	L140P	C (straight cut)	3286
2	1	3	7	L140P	C (straight cut)	3440
3	1	3	7	L140P	C (straight cut)	3266
4	1	3	7	L140P	C (straight cut)	3370
1	2	3	8	L120	C (straight cut)	3045
2	2	3	8	L120	C (straight cut)	3003
3	2	3	8	L120	C (straight cut)	3137
4	2	3	8	L120	C (straight cut)	3296
1	3	3	9	DKL 38-48	C (straight cut)	2644
2	3	3	9	DKL 38-48	C (straight cut)	2597
3	3	3	9	DKL 38-48	C (straight cut)	2532
4	3	3	9	DKL 38-48	C (straight cut)	2717

Table 5. Fertilizer trials involving four sources of nitrogen (UREA, SCU, USG, USG/UREA), two levels of nitrogen (low and high) and a control (no fertilizer) in a completely randomized design.

TREATMENT	REP	Source	Level	Control vs Treated	GYIELD
CONTROL	1	1	1	1	2.932
Low N UREA	1	2	2	2	4.528
Low N SCU	1	3	2	2	5.086
Low N USG	1	4	2	2	6.322
Low N USG/UREA	1	5	2	2	5.250
High N UREA	1	2	3	2	5.680
High N SCU	1	3	3	2	6.156
High N USG	1	4	3	2	6.164
High N USG/UREA	1	5	3	2	5.954
CONTROL	2	1	1	1	5.006
Low N UREA	2	2	2	2	4.258
Low N SCU	2	3	2	2	4.360
Low N USG	2	4	2	2	5.734
Low N USG/UREA	2	5	2	2	5.654
High N UREA	2	2	3	2	5.762
High N SCU	2	3	3	2	6.380
High N USG	2	4	3	2	6.730
High N USG/UREA	2	5	3	2	5.796
CONTROL	3	1	1	1	3.008
Low N UREA	3	2	2	2	5.710
Low N SCU	3	3	2	2	5.417
Low N USG	3	4	2	2	6.012
Low N USG/UREA	3	5	2	2	5.316
High N UREA	3	2	3	2	5.648
High N SCU	3	3	3	2	6.528
High N USG	3	4	3	2	6.944
High N USG/UREA	3	5	3	2	5.934

Table 6. Barley and Oat variety trial in a RCB design.

Crop	Within_Barley	Within_Oat	Variety	Block	% beta-glucan
Barley	Azure	.	Azure	1	4.69
Barley	Waxy Azure	.	Waxy Azure	1	5.42
Barley	Waxy Hull-less Azure	.	Waxy Hull-less Azure	1	6.47
Barley	Bowman	.	Bowman	1	4.32
Barley	Hull-less Bowman	.	Hull-less Bowman	1	4.43
Barley	Waxy Bowman	.	Waxy Bowman	1	5.72
Barley	Waxy Hull-less Bowman	.	Waxy Hull-less Bowman	1	4.82
Barley	WHASB	.	WHASB	1	5.55
Barley	Wanubet	.	Wanubet	1	6.52
Oat	.	Dumont	Dumont	1	5.61
Oat	.	Kelsey	Kelsey	1	4.54
Oat	.	Mariaon	Mariaon	1	5.91
Oat	.	Moore	Moore	1	5.2
Oat	.	Newdak	Newdak	1	4.67
Oat	.	Otana	Otana	1	5.09
Oat	.	Porter	Porter	1	4.86
Oat	.	Premeir	Premeir	1	6.22
Oat	.	Robert	Robert	1	4.74
Oat	.	Valley	Valley	1	5.72
Barley	Azure	.	Azure	2	4.52
Barley	Waxy Azure	.	Waxy Azure	2	5.26
Barley	Waxy Hull-less Azure	.	Waxy Hull-less Azure	2	6.19
Barley	Bowman	.	Bowman	2	4.37
Barley	Hull-less Bowman	.	Hull-less Bowman	2	4.45
Barley	Waxy Bowman	.	Waxy Bowman	2	5.25
Barley	Waxy Hull-less Bowman	.	Waxy Hull-less Bowman	2	4.84
Barley	WHASB	.	WHASB	2	5.7
Barley	Wanubet	.	Wanubet	2	6.48
Oat	.	Dumont	Dumont	2	5.34
Oat	.	Kelsey	Kelsey	2	4.51
Oat	.	Mariaon	Mariaon	2	6.27
Oat	.	Moore	Moore	2	5.59
Oat	.	Newdak	Newdak	2	4.66
Oat	.	Otana	Otana	2	5.21
Oat	.	Porter	Porter	2	4.77
Oat	.	Premeir	Premeir	2	6.03
Oat	.	Robert	Robert	2	4.59
Oat	.	Valley	Valley	2	5.23
Barley	Azure	.	Azure	3	4.37
Barley	Waxy Azure	.	Waxy Azure	3	5.23

Barley	Waxy Hull-less Azure	.	Waxy Hull-less Azure	3	6.1
Barley	Bowman	.	Bowman	3	4.03
Barley	Hull-less Bowman	.	Hull-less Bowman	3	4.4
Barley	Waxy Bowman	.	Waxy Bowman	3	5.37
Barley	Waxy Hull-less Bowman	.	Waxy Hull-less Bowman	3	5.21
Barley	WHASB	.	WHASB	3	6.33
Barley	Wanubet	.	Wanubet	3	6.12
Oat	.	Dumont	Dumont	3	4.75
Oat	.	Kelsey	Kelsey	3	4.89
Oat	.	Mariaon	Mariaon	3	5.63
Oat	.	Moore	Moore	3	4.93
Oat	.	Newdak	Newdak	3	4.53
Oat	.	Otana	Otana	3	5.9
Oat	.	Porter	Porter	3	4.4
Oat	.	Premeir	Premeir	3	6
Oat	.	Robert	Robert	3	4.76
Oat	.	Valley	Valley	3	4.9

Table 7. Wheat 3 factor split plot.

Plot	Y	X	Rep	Block	W_Plot	S_Plot	Fertility	Fungicide	Timing	Yield
1	6	1	1	1	1	1	Reduced	No Fungicide	Late	29.2
2	6	2	1	1	1	2	Reduced	Fungicide	Early	34.4
3	6	3	1	1	4	3	Reduced	No Fungicide	Early	29.6
4	6	4	1	1	1	4	Reduced	Fungicide	Late	43.5
5	5	1	1	1	2	1	Standard	Fungicide	Early	46.7
6	5	2	1	1	2	2	Standard	Fungicide	Late	60.8
7	5	3	1	1	2	3	Standard	No Fungicide	Late	46.2
8	5	4	1	1	2	4	Standard	No Fungicide	Early	50.0
9	4	1	2	2	1	1	Standard	No Fungicide	Late	43.2
10	4	2	2	2	1	2	Standard	Fungicide	Late	57.2
11	4	3	2	2	1	3	Standard	No Fungicide	Early	45.4
12	4	4	2	2	1	4	Standard	Fungicide	Early	54.9
13	3	1	2	2	2	1	Reduced	No Fungicide	Early	34.6
14	3	2	2	2	2	2	Reduced	Fungicide	Late	39.0
15	3	3	2	2	2	3	Reduced	Fungicide	Early	36.2
16	3	4	2	2	2	4	Reduced	No Fungicide	Late	37.3
17	2	1	3	3	1	1	Reduced	Fungicide	Late	53.2
18	2	2	3	3	1	2	Reduced	Fungicide	Early	46.5
19	2	3	3	3	1	3	Reduced	No Fungicide	Late	35.9
20	2	4	3	3	1	4	Reduced	No Fungicide	Early	43.7
21	1	1	3	3	2	1	Standard	No Fungicide	Late	45.5
22	1	2	3	3	2	2	Standard	Fungicide	Late	60.5
23	1	3	3	3	2	3	Standard	Fungicide	Early	52.5
24	1	4	3	3	2	4	Standard	No Fungicide	Early	52.1
25	6	5	4	4	1	1	Standard	No Fungicide	Late	35.0
26	6	6	4	4	1	2	Standard	No Fungicide	Early	33.6
27	6	7	4	4	1	3	Standard	Fungicide	Early	42.2
28	6	8	4	4	1	4	Standard	Fungicide	Late	62.0
29	5	5	4	4	2	1	Reduced	No Fungicide	Late	33.6
30	5	6	4	4	2	2	Reduced	No Fungicide	Early	36.1
31	5	7	4	4	2	3	Reduced	Fungicide	Early	33.5
32	5	8	4	4	2	4	Reduced	Fungicide	Late	41.3
33	4	5	5	5	1	1	Reduced	No Fungicide	Late	36.0
34	4	6	5	5	1	2	Reduced	Fungicide	Early	35.8
35	4	7	5	5	1	3	Reduced	Fungicide	Late	34.2
36	4	8	5	5	1	4	Reduced	No Fungicide	Early	40.4

37	3	5	5	5	2	1	Standard	Fungicide	Late	49.6
38	3	6	5	5	2	2	Standard	No Fungicide	Early	38.3
39	3	7	5	5	2	3	Standard	No Fungicide	Late	42.5
40	3	8	5	5	2	4	Standard	Fungicide	Early	59.4
41	2	5	6	6	1	1	Standard	No Fungicide	Early	49.6
42	2	6	6	6	1	2	Standard	Fungicide	Late	57.2
43	2	7	6	6	1	3	Standard	Fungicide	Early	49.0
44	2	8	6	6	1	4	Standard	No Fungicide	Late	58.5
45	1	5	6	6	2	1	Reduced	No Fungicide	Early	38.7
46	1	6	6	6	2	2	Reduced	Fungicide	Early	46.2
47	1	7	6	6	2	3	Reduced	No Fungicide	Late	42.4
48	1	8	6	6	2	4	Reduced	Fungicide	Late	58.3

Table 11. Orchard grass data from two locations in Ontario.

Plot	Entry	Block	Location	Yield
1	E1	1	Elora	9.5
1	E1	1	Thunder Bay	7.4
2	E2	1	Elora	9.3
2	E2	1	Thunder Bay	8
3	E3	1	Elora	9.3
3	E3	1	Thunder Bay	7.9
4	L1	1	Elora	7.8
4	L1	1	Thunder Bay	7.1
5	L2	1	Elora	8.8
5	L2	1	Thunder Bay	7.7
6	L3	1	Elora	7.9
6	L3	1	Thunder Bay	7.6
7	L4	1	Elora	9.4
7	L4	1	Thunder Bay	9
1	E1	2	Elora	9.5
1	E1	2	Thunder Bay	7.4
2	E2	2	Elora	10
2	E2	2	Thunder Bay	7.4
3	E3	2	Elora	10.2
3	E3	2	Thunder Bay	7.2
4	L1	2	Elora	7.6
4	L1	2	Thunder Bay	7.8
5	L2	2	Elora	8.7
5	L2	2	Thunder Bay	7
6	L3	2	Elora	9
6	L3	2	Thunder Bay	6
7	L4	2	Elora	9.3
7	L4	2	Thunder Bay	7.1
1	E1	3	Elora	9.3
1	E1	3	Thunder Bay	7
2	E2	3	Elora	9.4
2	E2	3	Thunder Bay	7.6
3	E3	3	Elora	9.1
3	E3	3	Thunder Bay	6.6
4	L1	3	Elora	8.4
4	L1	3	Thunder Bay	6
5	L2	3	Elora	9.3
5	L2	3	Thunder Bay	7.8
6	L3	3	Elora	8.6
6	L3	3	Thunder Bay	6.6
7	L4	3	Elora	9.6

7	L4	3	Thunder Bay	7.6
1	E1	4	Elora	9.7
1	E1	4	Thunder Bay	7.7
2	E2	4	Elora	10.3
2	E2	4	Thunder Bay	7
3	E3	4	Elora	10.7
3	E3	4	Thunder Bay	6.8
4	L1	4	Elora	9.3
4	L1	4	Thunder Bay	7
5	L2	4	Elora	11.1
5	L2	4	Thunder Bay	7.5
6	L3	4	Elora	7.5
6	L3	4	Thunder Bay	6.3
7	L4	4	Elora	10.2
7	L4	4	Thunder Bay	7.6